mgs | The Manchester Grammar School

Founded 1515

Entrance Examination 2013
Arithmetic Paper 2

1 Hour

Do not open this booklet until told to do so

Calculators may not be used

Write your names, school and candidate number in the spaces provided at the top of this page.

For each question, show all your working in full, as this will be marked, and then write your answer clearly in the space provided.

You have 1 hour for this paper which is worth 80 marks.

1. Complete the following sentences with the most appropriate metric unit

(a) Andy is 183 _____ tall

| 1a | |

(b) Our football pitch is 80 _____ long

| 1b | |

(c) My birthday cake weighs 700 _____
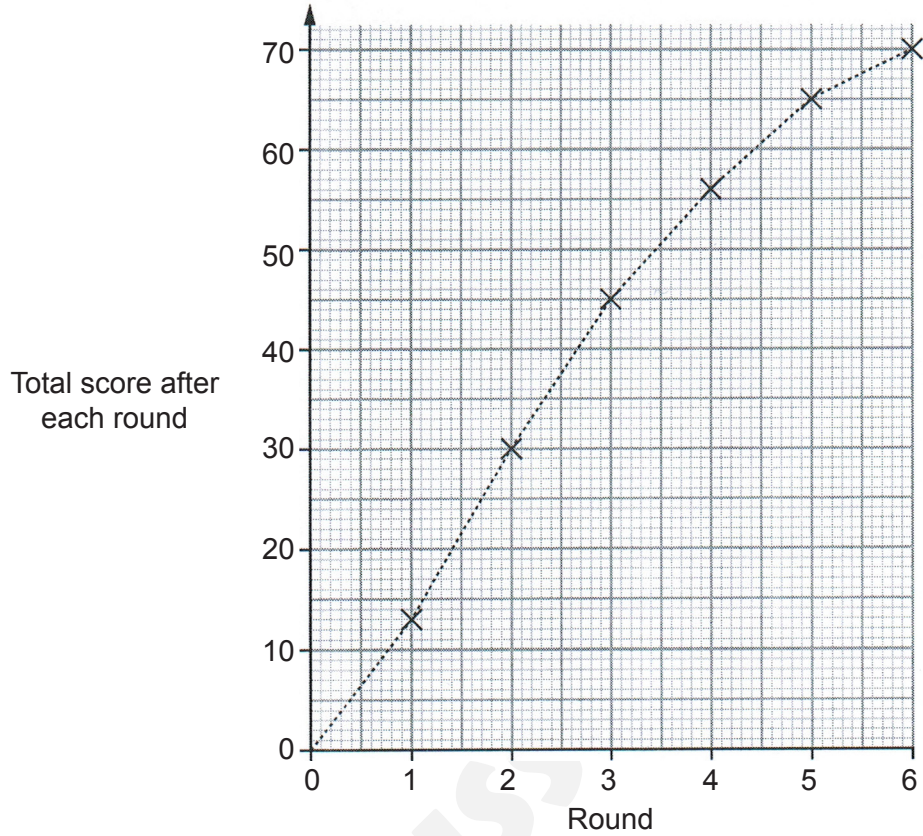
| 1c | |

(d) The area of a piece of lined writing paper is 600 _____

| 1d | |

(e) My water bottle contains 1.5 _____ of water

| 1e | |

[5 marks]

2. A game of counters has six rounds. In each round of the game, the players score points which are then added to their total score.

The graph below shows Claire's total score after each round of the game.

(a) How many points did Claire score in round 1?

2a

(b) How many points did Claire score in round 4?

2b

(c) Which was Claire's most successful round in which she scored most points?

2c

Nigel also plays the game and his scores are recorded. When the points showing his total scores are connected, the graph of his total scores after each round is a single straight line.

(d) What can you say about the number of points Nigel scored in each round?

2d

[5 marks]

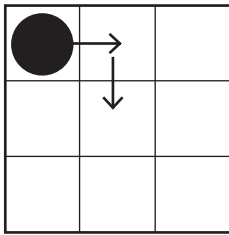3.   A piece in a game can move in four different ways:

Lown - one space to the left and one space down

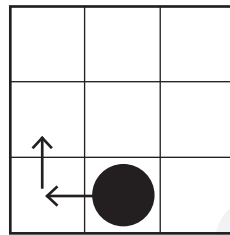Rown – one space to the right and one space down

Lup – one space to the left and one space up
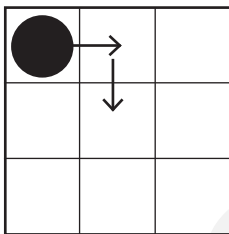
Rup – one space to the right and one space up

Rown:

Lup:

These moves can be combined to reach different positions on a board. For example on a 3 x 3 board, two Rowns would move the piece from the top left corner to the bottom right corner:
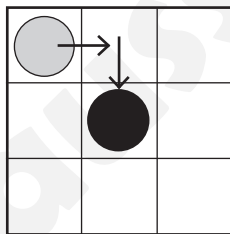
Start: After 1 Rown: After 2 Rowns:
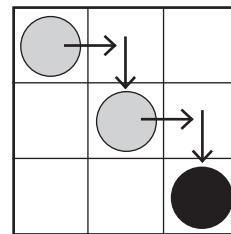
In parts (a) and (b), on the diagram alongside the question, show where the counter will be after the moves given. In each case the starting position is shown by the grey circle.

(a)   a Rup followed by a Lup   diagram for (a)
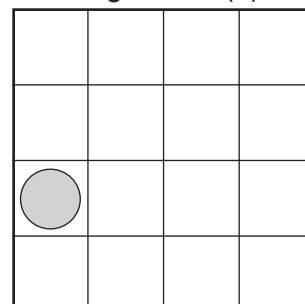
diagram for (b)

(b) 2 Lowns followed by a Lup
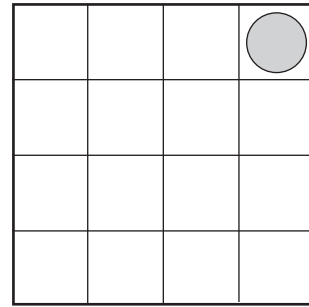
In parts (c) and (d) state the move which is the **<u>opposite</u>** of the given move.

(c) Lup

3c	

(d) Rown

3d	

(e) List a sequence of **<u>TWO</u>** moves which would take you from the light counter to the dark counter.

3e	then

[5 marks]

4. One of the ways to check if a number is divisible by 11 is by alternately subtracting and adding its digits as follows

Consider the digits in 121. Since 1 - 2 + 1 = 0 we can say that 121 is divisible by 11
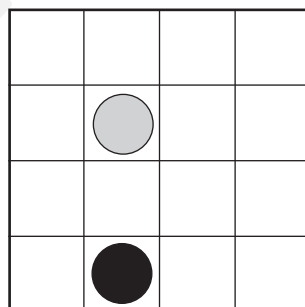
For the number 16918, the digits combine as 1 - 6 + 9 - 1 + 8 = 11, which is a multiple of 11 so 16918 is divisible by 11

However for 147, 1 - 4 + 7 = 4, which is not in the 11 times table, so 147 is NOT divisible by 11.

Neither is 3276 as 3 - 2 + 7 - 6 = 2, which is also not a multiple of 11

By applying this process of subtracting and adding, answer the following questions

(a) Show that 62557 is divisible by 11

4a	

(b) Can 138251 be divided by 11 and give no remainder? (Give your answer as either Yes or No)

4b	

(c) In the number 74n130 what value does the digit n have to be, if the number is divisible by 11?

4c	n =

(d) In the number 83a4b9 , the digit b is three times digit a. Find values for a and b if the number is divisible by 11.

4d	a =
	b =

[5 marks]

5. The ancient "Viva" tribe of Central America combined numbers in a particular way. Their method used the following symbol (.) and worked like this

a (.) b = 5 times a plus b (ie 5 x a + b)

so 2 (.) 7 = 5 x 2 + 7 = 17 and 9 (.) 3 = 5 x 9 + 3 = 48

Work out what the answer would be for these Viva calculations

(a) 3 (.) 4 =

5a	

(b) 5 (.) 3 =

5b	

Work out what value the letter would have to have to give the following results in these Viva problems

(c) If m (.) 2 = 37, what number must m be?

5c	m =

(d) If n (.) 4 = 8 (.) n, what number must n be?

5d	n =

[5 marks]

6. When a ball is dropped, the relationship between the speed (S) of the ball in m/s and the distance (d) in m it has fallen is

$$S^2 = 20 \times d \quad \text{where } S^2 = S \times S$$

So after dropping 5m, $S^2 = 20 \times 5 = 100$ and therefore the speed would be 10m/s as $10 \times 10 = 100$.

(a) Find the speed when the ball has dropped 45m

6a	m/s

(b) When the speed is 40 m/s, how far has the ball fallen?

6b	m

(c) How much **further** would the ball have to fall to be travelling at twice the speed that the ball had in part (b)?

6c	m

[5 marks]

7. In a **"prod sequence"**, a number in the sequence is the **product** of some of the previous terms. **So in a "2 prod sequence" we start with 2 numbers and then each number that follows after is the product of the previous 2 numbers.**

For example   1, 2, 2, 4, 8 ……..

Similarly, where we start with the three numbers 1, 2 and 3 then a "3 prod sequence" would be

               1, 2, 3, 6, 36, 648 ……

Fill in the missing numbers in the following sequences

(a) 2 prod sequence   2,   3,   _____,   _____

7a	,

(b) 3 prod sequence   2,   3,   3,   _____,   _____

7b	,

(c) 4 prod sequence   2,   3,   5,   _____,   210

7c	

(d) 2 prod sequence

2,   _____,   _____,   _____,   500

7d	then then

(e) 4 prod sequence

2,   2,   _____,   7,   _____,   3528

7e	and

[10 marks]

8. Following independence, the new country of Oldhallia is trying to design its national flag. The government decide that their flag will consist of two colours.

For example:

Blue	Green

because international rules state that the flag may not be all one colour and the rules also state that they are only allowed to pick from a certain number of colours.

The country always chooses what colour to put in the left section first. When calculating how many possible choices that they have for their flag; the following system is adopted. So with seven possible colours the choices would be

Any of the 7 colours	Any of the 6 remaining colours

Which gives a total number of possible flags = 7 x 6 = 42

(a) How many possible flags are there if they can choose from 10 colours?

8a	

(b) If there are a total of 12 flags, how many colours did they have to choose from?

8b	

(c) If the international rules were relaxed to allow flags of all one colour as well, how many possible flags would there be choosing from 6 colours?

8c	

If the country wants to make a flag with three colours, the rules state that they may not have two sections of the same colour next to each other.

If there were 4 choices of colour the calculation would be:

Any of the 4 colours	Any of the 3 remaining colours	Any of the 3 colours which are different from the middle

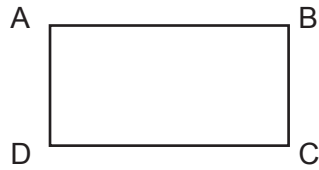**which gives a total number of possible flags = 4 x 3 x 3 = 36**

(d) How many flags are there if they can choose from 10 colours?

8d	

(e) If there are a total of 150 possible flags, how many colours did they have to choose from?

8e	

[10 marks]

Please turn over

9.   Here is a diagram of a rectangle with corners labelled ABCD.

A ┌──────────────┐ B
  │   │
 │   │
D └──────────────┘ C

When the rectangle is reflected about the vertical line shown, the corners end up in the positions indicated on the picture below. **We will call this change M1.**

B ┌──────┆───────┐ A
  │   ┆   │
C └──────┆───────┘ D
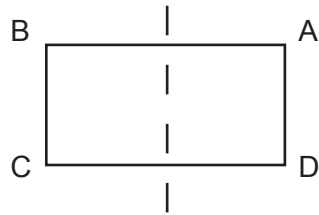
If the original rectangle is reflected about the horizontal line shown, the corners end up in the positions on the picture underneath. **This change will be called M2.**

D ┌──────────────┐ C
─ ┆ ─ ─ ─ ─ ┆ ─
A └──────────────┘ B
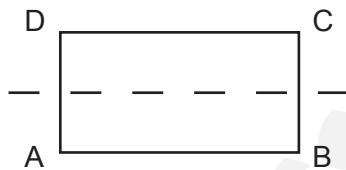
If the orginal rectangle is rotated through 180° about its centre O then the corners end up in the positions on the picture below. **This change will be known as R.**

C ┌──────────────┐ D
  │   O.   │
B └──────────────┘ A
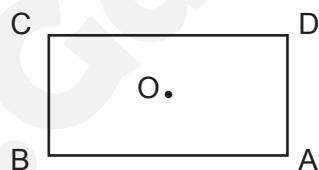
M1, M2 and R are known as transformations of the original rectangle

(a)   In the space below, draw and label the rectangle after performing transformation R followed by M1 **on the original rectangle.**

(b)   In the space below, draw and label the rectangle after performing transformation M1 followed by M2 **on the original rectangle.**

(c)   Using your answers to parts (a) and (b) and any other diagrams you may wish to draw, **complete the table below,** showing how **the original rectangle** is changed when a pair of transformations are performed.

2nd transformation

	R	M1	M2
R	I		
M1		I	
M2			I

1st transformation

where I indicates that a pair of transformations have taken you back to the original rectangle

(d)   Either by using your table or otherwise, find the **single** transformation which would take you back to the start having performed M1 then M2?

9d	

(e)   Either by using your table or otherwise, find the **single** transformation which would take you back to the start after having performed M1 then R?

9e	

[10 marks]

Please turn over

10. Ollie the sports coach wanted to take groups of children and their parents to see a range of sporting fixtures. When he made enquiries about the tickets there were always two types of ticket available, one for adults and one for children.

(a) For football at Rovers ground an adult ticket costs £15 and a child's ticket costs £4. Find the total cost for 20 adults and 12 children.

10a £ _____

(b) For a rugby match, 30 adults and 16 children paid £408 to see the game. A child's ticket cost £3. Work out the cost of one adult ticket.

10b £ _____

(c) 50 adults and 40 children went to the velodrome for the cycling championships. The total bill for the tickets was £760. An adult ticket cost three times as much as a child's ticket. Work out the cost of an adult ticket.

10c £ _____

(d) At the swimming tournament a child's ticket cost £6 less than an adult ticket. The total cost of the tickets for 25 adults and 30 children was £370. Work out the cost of one child ticket.

10d £ _____

[10 marks]

11. When I throw a green dice and a red dice together there are 36 number outcomes. **Note that green1, red2 and green2, red1 are different outcomes.**

   You may find it helpful to complete the grids below to help you, although completing the grids does not gain extra marks. The grids will be useful in answering the questions below

			Red			
Sum (+)	1	2	3	4	5	6
1	2		4			7
2		4			7	
3	4		6	7		9
4		6			9	
5		7		9		11
6	7	8	9		11	

(Green)

			Red			
Product (x)	1	2	3	4	5	6
1	1	2	3		5	6
2		4	6		10	
3	3		9		15	18
4	4			16		
5		10	15			
6	6					36

(Green)

			Red			
Difference (-)	1	2	3	4	5	6
1	0	1			4	5
2		0			3	
3			0	1		3
4				0		
5	4	3		1	0	1
6		4			1	0

(Green)

(a) How many sums of the two numbers are factors of 24?

11a [ ]

(b) How many of the products are even?

11b [ ]

(c) How many of the differences are prime numbers (remember 1 is not a prime number)

11c [ ]

(d) For what fraction of the throws is the green score greater than the red score?

11d [ ]

(e) What is the probability that the sum of the two numbers is more than six (6)?

11e [ ]

[10 marks]

The End